did not interact with each other. Demagnetizing factors were disregarded for the sake of simplicity.

Let us consider a resonant cavity containing the nonoriented ferrite. A biasing field is applied in a direction perpendicular to the RF magnetic field in the cavity. The resonant frequency of each crystallite will be determined by its anisotropy field, the biasing field, and the angle ψ its C axis makes with the biasing field. At one particular angle ψ_r for a given biasing field, the resonant frequency of the crystallite will be exactly the same as the test frequency and have the maximum interaction with the cavity. As the angle of the C axis departs from ψ_r, the resonant frequency becomes increasingly different from the test frequency and the interaction with the cavity decreases. We calculate the angles ψ_1, and ψ_2 between which a crystallite must lie in order that its resonant frequency will differ from the test frequency by no more than a chosen amount. All crystallites within this angle are presumed to absorb energy equally; all other crystallites are presumed not to absorb any energy. Let Ω be the solid angle subtended between the cones defined by ψ_1 and ψ_2. The loss term of magnetic susceptibility is proportional to Ω and therefore a plot of Ω vs biasing field is a plot of the relative value of χ'', the loss term of the susceptibility, vs biasing field. The line width is readily determined from such a curve.

Discussion

A plot of χ'' (relative) for a nonoriented uniaxial ferrite is shown in Fig. 1, and plots for nonoriented planar ferrites are shown in Fig. 2. The abscissa in both figures is the shifted biasing field H_0-H_r where H_0 is the applied biasing field and H_r is the biasing field required for ferromagnetic resonance for a crystallite whose easy direction is parallel to the biasing field (for the uniaxial ferrite) or whose easy plane is parallel to the biasing field (for the planar ferrite). Ha is the magnetic anistropy field.

A comparison of Figs. 1 and 2 shows that the line width of the nonoriented uniaxial ferrite is indeed very much larger than that of the planar ferrite. The most suitable comparison is between curve I of Fig. 2 and the curve in Fig. 1, since both have approximately the same value of anisotropy field and the same value of H_r. We note that the line width of the uniaxial ferrite is almost five times that of the planar ferrite.

The relatively narrow line width of nonoriented planar ferrites has been confirmed experimentally. Schlömann[9] has reported a line width of 500 oersteds for a nonoriented zinc Y. Of six nonoriented planar ferrites measured here, three had line widths of 1500 oersteds or less. It is interesting to find that completely nonoriented planar ferrites can have line widths narrower than the narrowest line width that has up to now been obtained with oriented polycrystalline uniaxial ferrites.

[9] E. Schlömann and R. Jones, "Ferromagnetic resonance in polycrystalline ferrites with hexagonal crystal structure," *J. Appl. Phys.*, Suppl. to vol. 30, pp. 177–178; April, 1959.

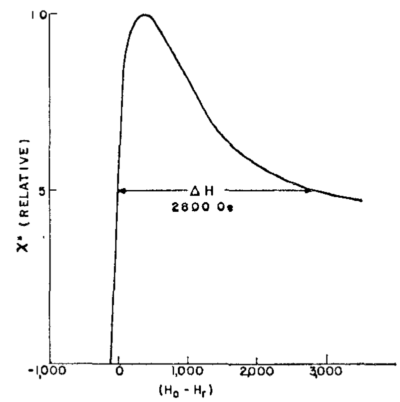

Fig. 1—Plot of χ'' (relative) vs shifted biasing field for a uniaxial ferrite with $H_a=8500$ oersteds, $\omega/\gamma=10{,}500$ oersteds, $H_r=2000$ oersteds.

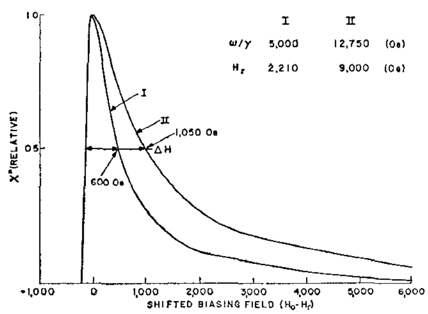

Fig. 2—Plot of χ'' (relative) vs shifted biasing field for planar ferrite with $H_a=9000$ oersteds, $\omega/\gamma=5000$ and 12,750 oersteds.

An understanding as to why the line width of nonoriented uniaxial ferrites is so much greater than that of nonoriented planar ferrites can be obtained from the following reasoning. The magnitude of Ω, and hence the magnitude of the loss term of the susceptibility, is proportional to two factors. Factor 1 is the magnitude of $|\psi_1-\psi_2|$ and factor 2 is the solid angle, Ω_Δ subtended between the cones defined by ψ_r and $\psi_r+\Delta\psi_r$ where $\Delta\psi_r$ is a small increase in ψ_r. The terms Ω, ψ_r, ψ_1 and ψ_2, have been defined previously.

Let us consider the variation of the two factors as a function of biasing field. Factor 1 is maximum when the biasing field is such that crystallites that are at ferromagnetic resonance are those whose easy direction of magnetization, or easy plane of magnetization (as applicable) are parallel to the biasing field. This biasing field has previously been designated as H_r. Factor 1 decreases as the biasing field is increased beyond H_r. Thus factor 1 is relatively large when ψ is close to 0° for the uniaxial ferrites, and close to 90° for the planar ferrites.

The solid angle subtended between the cones defined by ψ_r and $\psi_r+\Delta\psi_r$ is proportional to sin ψ_r. Thus factor 2 is small for biasing fields close to H_r for uniaxial ferrites and increases as the biasing field is increased beyond H_r. In the case of planar ferrites, factor 2 is large for biasing fields close to H_r and decreases as the biasing field is increased beyond H_r.

Thus in the case of uniaxial ferrites, as the biasing field is increased beyond H_r, factor 1 decreases and factor 2 increases. This tends to reduce the dependence of Ω on H_r as the biasing field is increased beyond H_r and results in a relatively broad line width. In the case of the planar ferrites, however, both factors are large in the vicinity of H_r, and both decrease as the biasing field is increased beyond H_r. Thus, there is a relatively sharp peak of Ω in the vicinity of H_r, and this results in a relatively narrow line width.

Conclusions

Theoretical calculations show that nonoriented uniaxial ferrites have a much wider line width than that of nonoriented planar ferrites. Thus the imperfect orientation that will inevitably occur when processing oriented polycrystalline hexagonal ferrites, will have a greater effect on broadening the line width of uniaxial ferrites than that of planar ferrites. This explains at least part of the reason why oriented planar ferrites generally have a much narrower line width than that of oriented uniaxial ferrites. In fact, a number of completely nonoriented planar ferrites have been prepared which have a substantially narrower line width than the narrowest line width achieved so far with polycrystalline oriented uniaxial ferrites.

I. Bady
G. McCall
U. S. Army Electronics Research and Development Lab.
Fort Monmouth, N. J.

E-Plane 3-Port *X*-Band Waveguide Circulators*

When a circulator is used with a parametric amplifier or maser, the noise contribution of the circulator may be reduced by cooling it in liquid nitrogen or liquid helium. Compact devices are required to put in the dewar and, depending on the microwave frequency, a compromise may be necessary in choosing between a compact stripline circulator and a comparatively bulky H-plane waveguide circulator, because waveguide feeds will have lower loss than coaxial line. This problem may be eased by using a very compact E-plane waveguide circulator, as shown in Fig. 1(a).

It can be shown that the circulation bandwidth and performance of a lossless, nonreciprocal, symmetrical 3-port waveguide junction are dependent only on the frequency characteristic of the reflection coefficient. In a practical device the circulation may occur in opposite senses at various frequencies. These modes of circulation can be defined in terms of the microwave frequency, the value of applied magnetic field, the direction of circulation and the dimensions of the ferrite. Therefore, in principle, there are two stages in the development of a

* Received June 3, 1963.

3-port circulator: the search for a broad-band mode of circulation, followed by matching over the broad band. This communication describes an investigation which has shown that a broad-band mode is possible in an X-band E-plane 3-port waveguide junction. Yoshida[1] and Buchta[2] have previously demonstrated that narrow-band circulation is possible.

The static magnetic field of a circulator must be applied perpendicular to the plane of the junction, *i.e.*, in this case, perpendicular to the narrow walls of the waveguide. Since the RF magnetic field in the material must be perpendicular to the static field, the ferrite is placed on or near the narrow walls. This orientation is shown in Fig. 2(a) and is compared with the more usual H-plane orientation in Fig. 2(b). In both cases (for small sample sizes) the ferrite is irradiated by a plane-polarized wave and will cause an asymmetrical radiation pattern. In the first case the pattern will be in the z-x plane, Fig. 2(c), and in the second case it will be in the y-z plane, Fig. 2(d). A theoretical analysis of a practical E-plane device, *i.e.*, where the sample size is not small, would be complex because there is a variation of the RF field in the direction of the polarizing field. This is a complication which does not arise in the treatment of an H-plane circulator.

The ferrite used in all the experiments on this circulator was a manganese-magnesium ferrite, $4\pi M_s = 2200$ gauss, Mullard ferrite type $D5$.

Firstly, it was verified that a piece of ferrite placed in the center of the junction [Fig. 1(b)], where the RF magnetic field is substantially parallel to the static field, produced negligible circulation. This was checked with a disk 0.4 inch in diameter, 0.10 inch thick, and a short post, 0.20 inch in diameter, 0.40 inch long. Then a flat disk, 0.45 inch in diameter, 0.05 inch thick was placed against each narrow wall, Fig. 1(c) and poor circulation was observed over a broad band with an applied field of about 1500 oersteds. As the thickness was increased the performance improved and the operating bandwidth shifted to lower frequencies. With a thickness 0.10–0.15 inch, a mode of circulation in the reverse sense appeared at the upper end of the waveguide pass band and the value of static magnetic field required was less than 1000 oersteds. The performance obtained with disks 0.45 inch in diameter, 0.15 inch thick, in this upper mode is shown in Fig. 3. Then measurements were made using a thin rod of ferrite (0.2 inch in diameter, 0.9 inch long) which extended across the junction, Fig. 1(c), and poor circulation was again observed over a broad band. The fact that the part of the rod in the center of the junction provides very little circulation was confirmed by measurements on shorter rods attached to the narrow walls. With 0.20-inch diameter, the performance was not substantially improved by using posts longer than about 0.25 inch. These results are shown in Fig. 4, which displays the variation of the differential isolation with frequency. The differential isolation is the difference in power in the output port 2 and the isolated port 3. This can readily be obtained by monitoring either port and reversing the magnetic field. The performance obtained with the short thin posts was poor and the results of attempts to improve it by increasing the diameter to 0.27 inch are shown in Fig. 5. A single mode is maintained over a broad band only with thicknesses of 0.15 inch or less. With thicknesses greater than this, a mode of reverse circulation appears at the upper end of the waveguide pass band, as occurred with the thick disks.

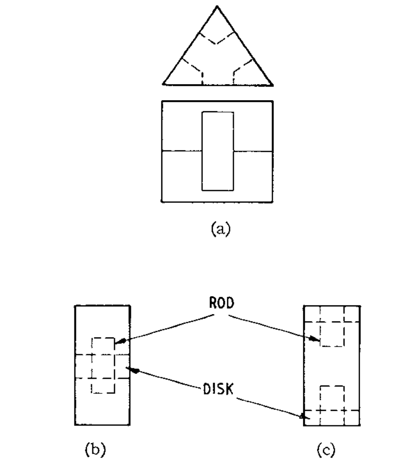

Fig. 1—(a) The E-plane 3-port waveguide circulator. (b) Negligible circulation. (c) Broad-band circulation.

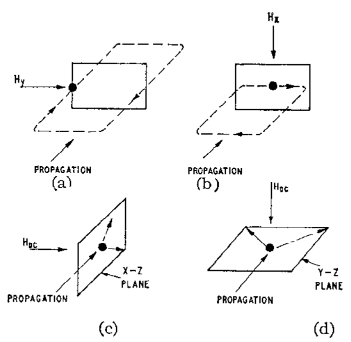

Fig. 2—The ferrite positions in waveguide circulators. (a) E-plane orientation. (b) H-plane orientation, with asymmetrical scattering in: (c) x-z plane, $\partial E/\partial y$, $\partial H/\partial y \neq 0$. (d) y-z plane, $\partial E/\partial x = \partial H/\partial x = 0$.

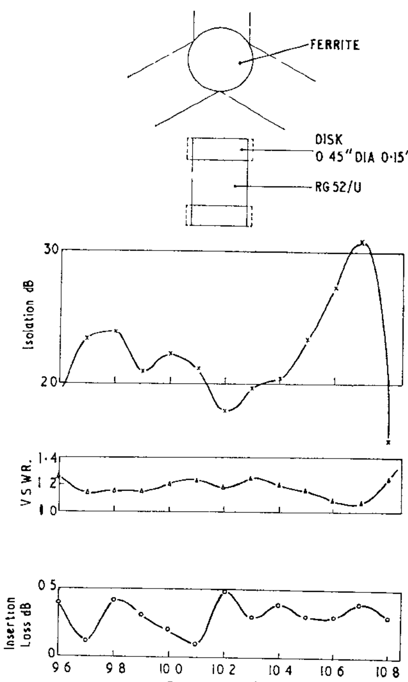

Fig. 3—Circulation obtained with disks 0.45 inch in diameter, 0.10 inch thick.

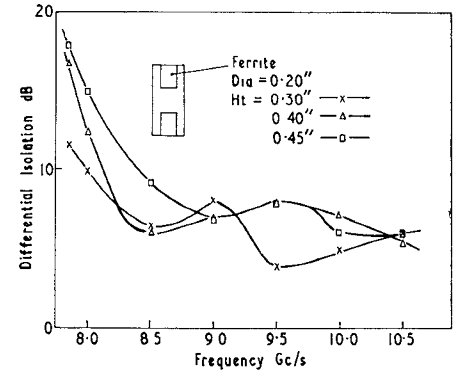

Fig. 4—Differential isolation obtained with short rods 0.20 inch in diameter. The ferrite near the center of the junction contributes little to the circulation.

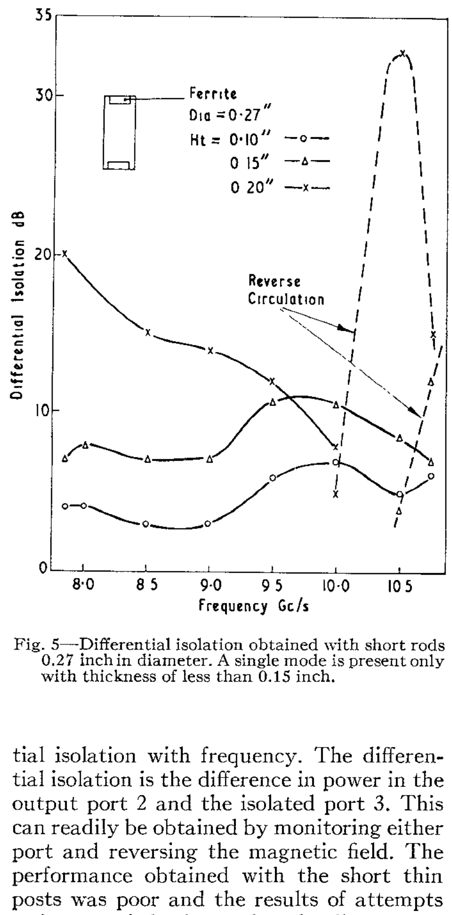

Fig. 5—Differential isolation obtained with short rods 0.27 inch in diameter. A single mode is present only with thickness of less than 0.15 inch.

Thus, a single broad-band mode of circulation can be found using either ferrite posts, provided that the diameter is less than 0.20 inch, or ferrite disks, provided that the thickness is less than 0.15 inch. The disks offer the better performance, and the frequency range of circulation may be optimized by adjusting the ferrite dimensions. The thicker the disk, or the larger its diameter, the lower the optimum frequency. How-

[1] S. Yoshida, "E-type T circulator," PROC. IRE (*Correspondence*), vol. 47, p. 208; November, 1959.

[2] G. Buchta, "X-Band Circulator" Philips Central Laboratory, Hamburg, Germany, private communication; February, 1963.

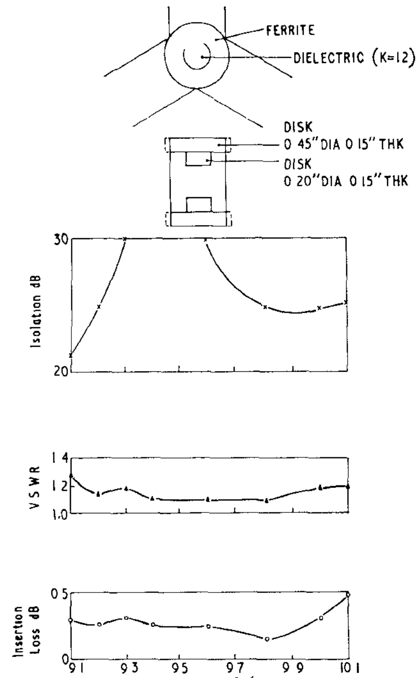

Fig. 6—Circulation obtained with ferrite disks and dielectric.

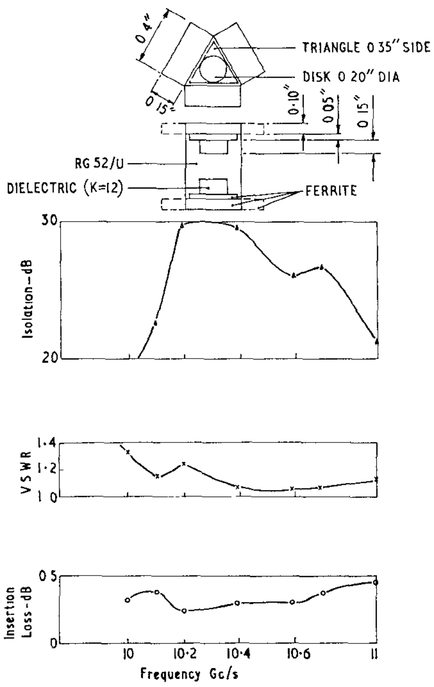

Fig 7—Circulation obtained with the triangular ferrite configuration and dielectric.

ever, where the thickness is greater than 0.10 inch the diameter must rapidly be reduced to approach 0.20 inch at a distance of 0.15 inch–0.20 inch from the narrow walls in order to prevent complex behavior.

Similar results were obtained with triangular pieces of ferrite instead of disks. With triangles, 0.4 inch side, 0.10 inch thick, a broad-band mode of circulation occurred in the frequency range 8–10 Gc. Increasing the thickness to 0.15 inch deteriorated that mode, shifted the frequency range down, and introduced a mode of opposite circulation at the higher frequencies. But the performance of the original mode was improved without introducing the higher one by using smaller triangles on top of the 0.4 inch side, 0.10 inch thick pieces. Thus, the behavior is similar to that of the disks, a convergent tapering of the ferrite towards the center of the junction being necessary to maintain a single broadband mode of circulation.

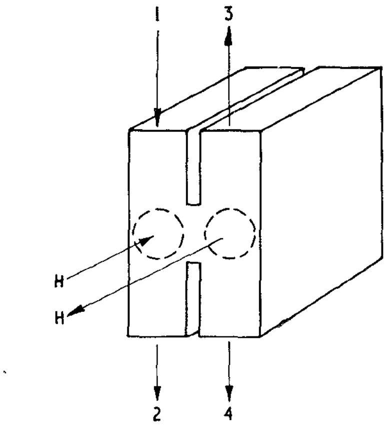

Fig. 8—Two oppositely polarized folded E-plane circulators forming a very compact 4-port circulator.

Two matching techniques were used to improve the performance of these devices. Adding dielectric on top of the ferrite, and placing small rectangular ferrite slabs against the sides of the triangular pieces are methods which can be used separately or together. Examples of both these configurations giving bandwidths of about 10 per cent are shown in Figs. 6 and 7. The addition of dielectric on top of the ferrite is the preferable method, since this will permit greater compactness, and a further reduction in volume may be possible if a reduced-height waveguide junction is used.

It has been established that a single mode of circulation does exist over the waveguide pass band using an E-plane 3-port junction with thin ferrite disks placed against the narrow walls. Compact circulators of this type may offer a realistic alternative to the stripline 3-port circulator. This is particularly relevant when a "straight-through" geometry is sought, as would be required in a dewar application. For this purpose two folded E-plane 3-port devices with opposite senses of polarization would form a very compact 4-port circulator, as shown in Fig. 8.

Finally, it is interesting to speculate on the fundamental difference between the E- and H-plane 3-port circulators. They depend respectively on the longitudinal and transverse components of the RF magnetic field which have opposite frequency dependence. This would suggest that the E-plane device would have a better performance at the lower end of the waveguide pass band, and the H-plane a better performance at the upper end.

L. E. Davis
S. R. Longley
Mullard Research Labs.
Redhill, Surrey, England

Comments on "Pulse Waveform Degradation Due to Dispersion in Waveguide"*

Recent work at The Hallicrafters Company concerned with the analysis of the transmission of pulsed electromagnetic energy through dispersive media has caused the writers to review the above work of R. S. Elliott.[1] In this review it was noted that (14) of that work contains an error (which has been brought to the attention of R. S. Elliott, who agrees that it does exist). This equation should read

$$F(t) = \frac{1}{\sqrt{2}}\sqrt{X^2 + Y^2}, \tag{1}$$

where

$$X = C(A_1) - C(A_2) \tag{2}$$

$$Y = S(A_2) - S(A_1) \tag{3}$$

with

$$C(A) = \int_0^A \cos\left(\frac{\pi}{2}y^2\right)dy$$

$$= \text{Cosine Fresnel Integral} \tag{4}$$

$$S(A) = \int_0^A \sin\left(\frac{\pi}{2}y^2\right)dy$$

$$= \text{Sine Fresnel Integral} \tag{5}$$

$$A_1 = \frac{x+1}{a\sqrt{\pi/2}} \tag{6}$$

$$A_2 = \frac{x-1}{a\sqrt{\pi/2}} \tag{7}$$

$$x = \frac{2t'}{T} \tag{8}$$

$$a = \frac{4}{T}\sqrt{BL} \tag{9}$$

$$t' = t - AL. \tag{10}$$

Eq. (1) has also been obtained independently and at about the same time by R. O. Brooks of the Raytheon Company.

The above is for an input pulsed carrier turned on at time $= -T/2$ and of duration T. For the same pulsed carrier, but turned on at time $t = 0$, the result is given by (1) with X and Y given by

$$X = C(A_1') - C(A_2') \tag{11}$$

$$Y = S(A_2') - S(A_1'), \tag{12}$$

where

$$A_1' = \frac{x}{a\sqrt{\pi/2}} \tag{13}$$

$$A_2' = \frac{x-2}{a\sqrt{\pi/2}}. \tag{14}$$

Numerical computations of (1) using (11) and (12) for the cases of $a = 0$, 0.032, 0.10, 0.32, 0.50, and 1.00 are shown in Fig. 1, and reveal that these shapes are practically identical to those of Elliott except for the large values of a. The computations were performed using the Fresnel Integral Tables of Pearcey,[2] and are tabulated in Table I.

* Received May 6, 1963.

[1] R. S Elliott, "Pulse waveform degradation due to dispersion in waveguides," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 254–257; October, 1957.

[2] T. Pearcy, "Table of the Fresnel Integral," Cambridge at the University Press, Cambridge, England, 1956.